

Sample Math Problems:

EXAMPLE PROBLEM #1:

In an apartment block is an elevator with two buttons.

- If the first button is pressed, we rise (up) 7 floors.
- If the second button is pressed, we descend 9 floors (down).

How should we press the buttons to get from the first floor to the 72nd floor?

Indicate the sequence of pressing the buttons.

EXAMPLE PROBLEM #2:

Anna, Bob, Cathy, Dawn, Echo are being lined up for a picture. How many ways can this be done so that Cathy and Dawn are NEVER next to each other?

EXAMPLE PROBLEM #3:

A school has 1000 lockers and they are all numbered consecutively. A student experiment is performed where each student will walk into the school one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with EVEN numbers. The third student will change all locker doors that are multiples of 3. (change means closing doors that are open and opening doors that are closed). The fourth student will change the position of all locker doors numbered with multiples of 4; the fifth student will change the position of the locker doors that are multiples of 5 and so on. After 1000 students have entered the school, which locker doors will be open?

EXAMPLE PROBLEM #4:

A Class of 45 students completed a survey on what pets they like. The choices were: Cats, Dogs, and Birds. Everyone liked at least one pet. 10 students liked Cats and Birds but not dogs, 6 students liked Cats and Dogs but not birds, 2 students liked Dogs and Birds but not Cats, 2 students liked all three pets, 10 students liked Cats only, 9 students liked Dogs only, 1 student liked Birds only. How many students surveyed do not like ANY of these pets?

Example Problem #5:

A rope of length 200 cm is cut into four pieces. Three of the pieces are used to form identical equilateral triangles with integer side lengths. The fourth piece is used to form a square with integer side lengths. Determine a possible side length for each triangle and square.

Solutions:

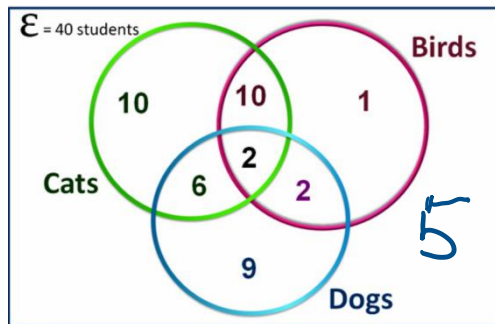
- The equation $7x - 9y = 71$ has one solution of $x = 14, y = 3$. Since 71 floors must be ascended, we must press the up buttons 14 times and the down button 3 times. There are other solutions that work!
- $(5 \times 4 \times 3 \times 2 \times 1) - (2 \times 4 \times 3 \times 2 \times 1) = 72$ Think of each place in the line up as a slot that needs to be filled. Consider the number of ways to fill each slot.

$$\underline{5 \times 4 \times 3 \times 2 \times 1} = 120 \text{ ways}$$

$$\underbrace{4 \quad 3 \quad 2 \quad 1}_{\text{treat as one}} \quad \left. \vphantom{\frac{4}{1}} \right\} \text{two ways (Cathy or Dawn first)}$$

- Only lockers that are numbered as PERFECT SQUARES will be open. Note that a perfect square has an odd number of divisors. Hint: if struggling, try this problem with a smaller number of lockers....say 10.

- Five of the students surveyed did not like any of the three pets.



- The perimeter of each figure is the length of the piece of rope used to form it. For each triangle, the length of rope is $3x$ and for the square the length of rope is $4y$. The total rope used is $3(3x) + 4y = 9x + 4y$. But the length of the rope is 200 cm. Therefore,

$$\begin{aligned} 9x + 4y &= 200 \\ 9x &= 200 - 4y \\ x &= \frac{4(50 - y)}{9} \end{aligned}$$

Since both x and y are integers, $4(50 - y)$ must be a multiple of 9. But 4 is not divisible by 9, so $50 - y$ must be divisible by 9.

There are five multiples of 9 between 0 and 50, namely 9, 18, 27, 36, and 45. So $50 - y = \{9, 18, 27, 36, 45\}$ and it follows that $y = \{41, 32, 23, 14, 5\}$. The corresponding values of x are computed in the chart below.

y	$4y$	$200 - 4y$	$x = \frac{200 - 4y}{9}$
41	164	36	4
32	128	72	8
23	92	108	12
14	56	144	16
5	20	180	20

When the side length of the square is 41 cm, the side length of each triangle is 4 cm; when the side length of the square is 32 cm, the side length of each triangle is 8 cm; when the side length of the square is 23 cm, the side length of each triangle is 12 cm; when the side length of the square is 14 cm, the side length of each triangle is 16 cm; and when the side length of the square is 5 cm, the side length of each triangle is 20 cm.